## Discrete Mathematics

Background

This topic deals with fundamentals of discrete mathematics (counting, relations, functions and partially ordered sets).

* **Mathematical Logic**

Prove identities/equalities/inequalities by induction

* **Counting**

multiplication principle, sumrule and the five basic formulas

pigeonhole principle

* **Relations and Digraphs**

Directed graphs, matrix of finite relation

Properties of relations on a set of pairs (reflexivity, irreflexivity,

symmetry, asymmetry, antisymmentry, transitivity)

transitive closure of a relation.

* **Functions**

one-to-one, onto, bijective, invertible and inverses

* **Order Relations and Structures**

Partial order (set + relation)

Isomorphic (two partial orders)

**Lesson 0**

Mathematical Induction

(From COS2601)

This essentially is the same as using a recursive definition to define a language. It uses a trick which allows you to prove a statement about an arbitrary number n by first proving it is true when n is 1 and then assuming it is true for n=k and showing it is true for n=k+1.

**Proof by induction:**

|  |  |
| --- | --- |
| Recursive Definition | Clause |
| Show n=1 is true (LHS = RHS) | Basis Clause |
| substitute k into formula (LHS) | Inductive Clause |
| Substitute k+1 into formula (RHS) | Extremal clause |

Example ASS 01 Q1a

Let be the statement

**Basis Clause**

Show that

is where

.

Therefore, is true

**Inductive Hypothesis**

Show that.

is where

Assume

**Extremal Clause**

If is true, then must also be true

Assume

But,

Therefore, by the induction hypothesis:

Thus, is true

Hence, is true

It then follows by mathematical induction that is true.

Example ASS 01 Q1a

Let be the statement

**Basis Clause**

Show that

is where

and

Therefore, is true

**Inductive Hypothesis**

Show that

is where

Assume

**Inductive Step**

If is true, then must also be true

Assume

But,

Therefore, by the induction hypothesis:

Re-write 4 as 3+1

Multiplying out

By regrouping

Remove from both sides

is true for all

Thus, is true

Hence, is true

It then follows by mathematical induction that is true for

Example

TODO: example with induction to prove divisibility

**Lesson 1**

Counting

Being able to count the number of ways something can happen.

**Product Rule (AND)**

* Procedure can be broken down into a sequence of tasks
* Number of outcomes of each task is
* different ways to perform a procedure

Example: If I have 4 different t-shirts and 3 different pairs of shorts, how many outfits do I have?

**Tree diagram**

* A visual strategy we can use to represent counting problems
* This is good if you are focused on a particular outcome

Example: If I have 4 different t-shirts and 3 different pairs of shorts, how many outfits do I have?

s1 t1s1

t1 s2 t1s2

s3 t1s3

s1

t1 s2

s3

s1

t1 s2

s3

s1

t1 s2

s3

**Sum Rule (OR)**

* If a task can be performed in one of or ways, then there are ways to perform the task.

Example: I want to take a trip to the beach. I can travel to one of 37 international beaches of one of 14 domestic beaches. How many beach destination choices do I have?

A close up of a logo

Description automatically generated**Subtraction Rule (Inclusion exclusion)**

* If a task can be done in either one of or ways, then the total number of ways to do the task is minus the number of ways that are common

Example: how many bitstrings of length 7 either start with 1 bit of end with 3 bits 000?

Number that start [1][ ][ ][ ][ ][ ][ ]

with 1

Number that end [ ][ ][ ][ ][0][0][0]

with 000

Number that start with 1 [1][ ][ ][ ][0][0][0]

and end in 000

**Division Rule**

* There are is ways to do a task if it can be done using a procedure that can be carried out in ways, where the corresponding outcomes per group.

Example: how many ways can I sit 6 people around a circular table where two seats are considered the same when each person has the same left and right neighbour?

***Set one person and then the rest***